V, I, R measurements: how to generate and measure quantities and then how to get data (resistivity, magnetoresistance, Hall).

590B

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September 28, 2009

Thermo- galvano-magnetic effects

Seebeck effect

Hall effect

Weak field magnetoresistance

A little bit of classification

Thermoelectric effects
Seebeck effect
Peltier effect (heating/cooling on current flow in contacts)
Thompson effect (heat/cooling in materials with current and T-gradient)

Thermo- Galvano- magnetic effects
(electrical and heat current carrying conductor in magnetic field)
Longitudinal and transverse with respect to the current

Longitudinal
Orbital magnetoresistance

Transverse
Hall effect
Nernst effect (transverse voltage with longitudinal heat current)
Ettingshausen effect (transverse temperature gradient)

And many more! (32 possible combinations!)

Seebeck Effect (1821)

The thermal gradient in an isolated conductor creates voltage difference (EMF)

Thomas Seebeck

Seebeck Effect, $\Delta V = V_C - V_H$

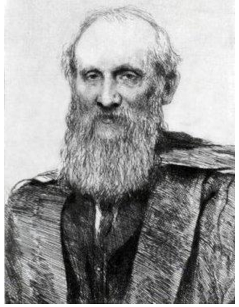
Seebeck Coefficient

$$S = \frac{d(\Delta V)}{dT}$$

S is positive when the direction of electric current is same as the direction of thermal current, or the potential of cold contact is higher than of the hot contact

Thompson Effect

When current flows in a homogeneous conductor with thermal gradient extra heat is absorbed/released



William Thomson (Lord Kelvin)

$I=0$ T **$T+\Delta T$, Q**

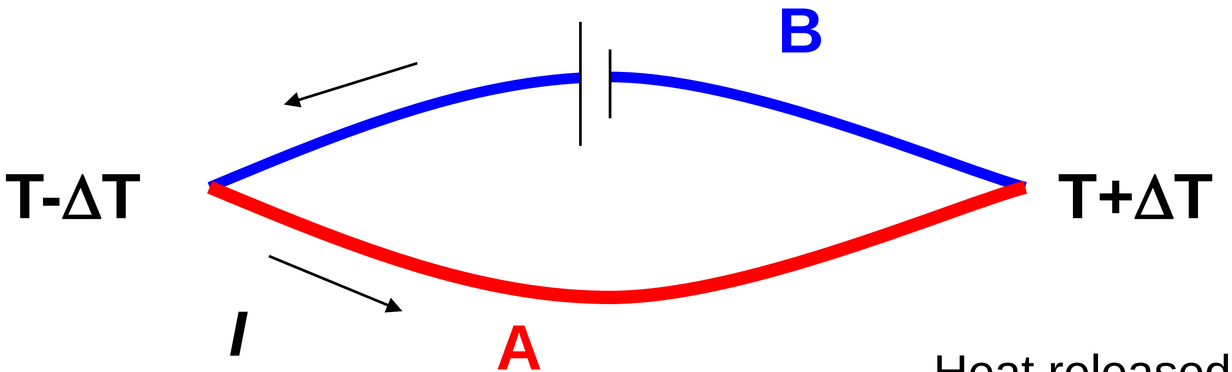
$I\neq 0$ T **$T+\Delta T$, $Q\pm\Delta Q$**

Thompson coefficient

$$\beta = \left| \frac{Q}{\Delta T} \right|$$

Peltier Effect (1834)

Heat absorption or release during current flow through a junction of dissimilar metals



Heat released/adsorbed
in the junction

$$\pi_{AB} = \pi_A - \pi_B = \frac{Q}{It}$$

time

In a contact π >0 if the direction of charge and heat currents coincide

π=ST

Mott formula for thermopower

$$S = \frac{\pi^2}{3}\frac{k_B{}^2 T}{e}\left(\frac{\partial \ln \sigma(\varepsilon)}{\partial \varepsilon}\right)_{\varepsilon=\varepsilon_F}$$

$$\sigma(\varepsilon) = e^2\tau(\varepsilon)\frac{dk}{4\pi^3}\delta(\varepsilon_F - \varepsilon(k))v(k)v(k)$$

$$S = \frac{\pi^2}{3}\frac{k_B{}^2 T}{e}\left[\underbrace{\left(\frac{\partial \ln \tau(\varepsilon)}{\partial \varepsilon}\right)_{\varepsilon=\varepsilon_F}}_{\text{transport}} + \underbrace{\frac{\int dk\delta(\varepsilon_F - \varepsilon(k))M^{-1}(k)}{\int dk\delta(\varepsilon_F - \varepsilon(k))v(k)v(k)}}_{\text{thermodynamic}}\right]$$

$$M_{ij}{}^{-1} = \pm\frac{1}{\hbar^2}\frac{\partial\varepsilon(k)}{\partial k_i \partial k_j}$$ Inverse of effective mass tensor

Difficult to understand in general case

Simple cases "good scattering"

Isotropic Fermi surface

Impurity scattering and T>>Θ_D

Here scattering does not have sharp energy dependence

$$\tau(\varepsilon) = \tau_0 \varepsilon^{\varsigma}$$

$$S = \frac{\pi^2}{3} \frac{k_B{}^2}{e} \frac{T}{E_F} \left(\frac{3}{2} + \varsigma \right)$$

Diffuse thermopower of free electron gas

The better the metal, $E_F\uparrow$ and $S\downarrow$

$$S \sim \frac{k_B}{e} \frac{k_B T}{E_F}$$

k_B/e=87 μV/K, characteristic thermopower unit
In metals S<<k_B/e

Semi-Intuitive approach (P. M. Chaikin)

$$S = \frac{\text{"heat"\_per\_carrier}}{\text{charge\_per\_carrier} \times T}$$

$$S = \frac{\text{entropy\_per\_carrier}}{q}$$

S=0 in states without entropy

- Superconductivity
- Sliding density waves
- Transport by filled Landau levels (quantum Hall effect)

Summation for several types of carriers

$$\boxed{S = \sum_i \left(\frac{\sigma_i}{\sum \sigma_i} \right) S_i}$$

Semiconductors

"Heat" of the carrier is a distance between
Fermi energy and valence and conduction band edges

$$"heat" = E - \mu \approx E_g - \mu \approx E_g / 2$$

$$S = \frac{E_g / 2}{eT} \approx \frac{k_B}{e} \frac{E_g}{2k_B T}$$

Since two types of carriers need to sum contributions,
Correct formula for intrinsic case

$$S = \frac{k_B}{e}\left[\frac{b-1}{b+1}\frac{E_g}{2k_B T} + \frac{3}{4}\ln\frac{m_e}{m_h}\right] \qquad b = \frac{\mu_e}{\mu_h}$$

$E_g >> k_B T$, $S \sim 1/T$, $S >> k_B/e = 87\ \mu V/K$

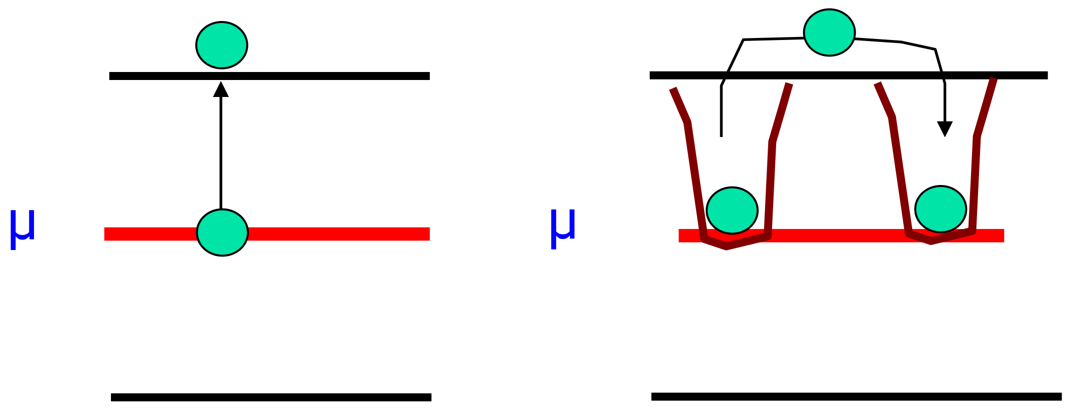
Thermopower of polarons

Interesting difference between resistivity and thermopower

In both semiconductor and polaron transport resistivity is activated

S=1/T in the first case, S~T in the second

Why? Energy diagram



Intrinsic Semiconductor

Polaron

In polaronic case
there is no change
of energy between
Initial and final states
S is small

Why bother measuring thermopower?

- Information on charge of carriers (not many ways to get!)
- Information of carrier density (indirect)
- Can distinguish cases of real gap and mobility gap
- Can distinguish intrinsic and impurity dominated transport
- Extreme sensitivity to superstructures and short range orderings

These produce anomalies in energy derivative of conductivity

$$\frac{\partial \sigma(\varepsilon)}{\partial \varepsilon}$$

Some useful materials for thermopower measurements

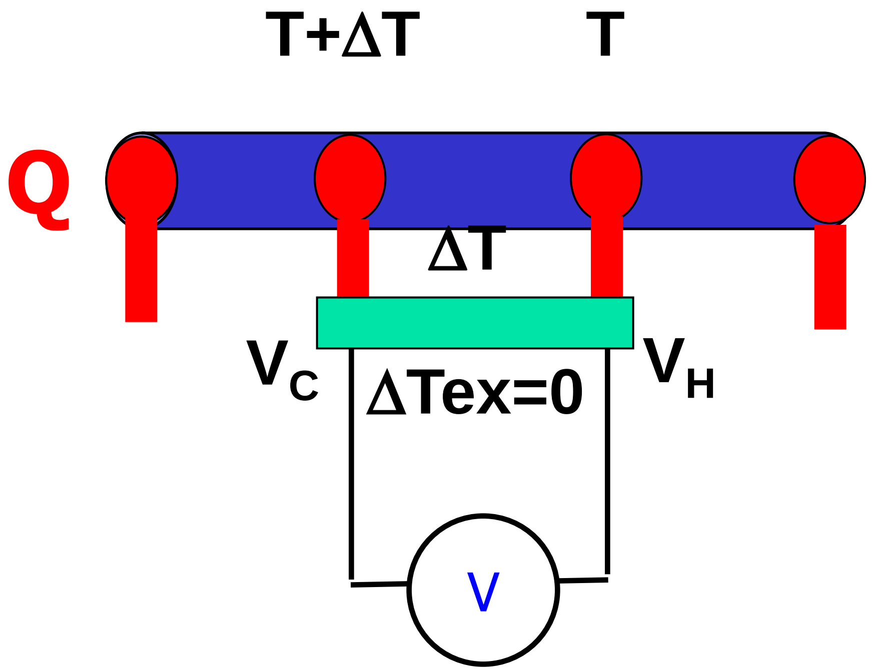
Supercondcutors, S=0

Lead (S<0.2 μV/K in all range <300K), frequently used for
High-T calibrations

Phosphor bronze (recent development)
S~0 (<1 μV/K at 300K, <0.1 μV/K at 20K and below)
Does not depend on magnetic field

Thermocouple materials
Constantan
-37 μV/K at room temperature
Chromel
Type E thermocouple

4-probe thermopower measurements



Best way: superconducting wires

The problem:
ΔT is not only in the sample, but in the measuring circuit

Inevitably pick up wire loop thermopower

Ways around:
Do not let gradient escape into external wires
Close the loop thermally

Inside the measuring loop use wires with well documented behavior
S=Ssample+Swire(addenda)

Apparatus for thermopower measurements on organic conductors*

P. M. Chaikin and J. F. Kwak

218 Rev. Sci. Instrum., Vol. 46, No. 2, February 1975

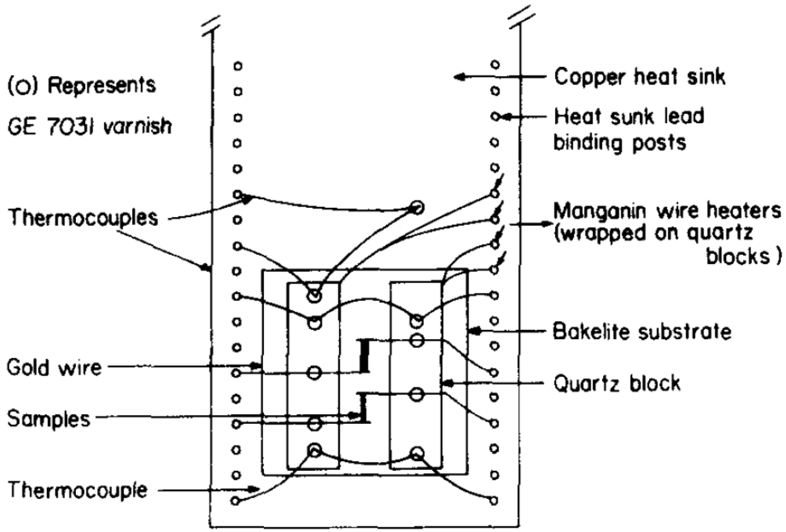


FIG. 1. Schematic representation of apparatus set up for thermopower measurements.

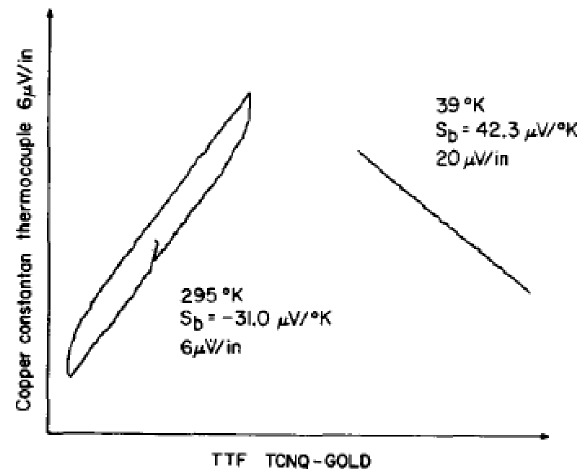


FIG. 2. Typical traces of thermocouple output vs thermopower differences between sample and gold leads.

Optimization parameter: fragile crystals
Problems: thermal drift
Eliminated by slow alternating thermal gradient

REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 72, NUMBER 5 MAY 2001

Low-frequency method for magnetothermopower and Nernst effect measurements on single crystal samples at low temperatures and high magnetic fields

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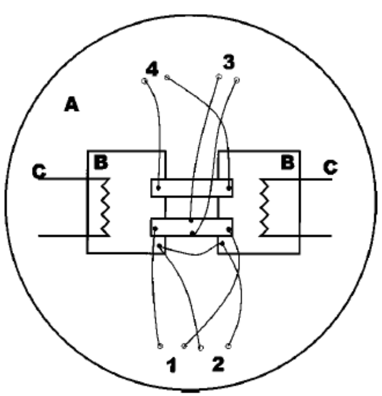

FIG. 1. Diagram of the measurement holder (the outer diameter of the cylindrical copper holder is 10 mm). A: Cu heat sink, B: quartz blocks, and C: heaters. 1: thermopower leads of sample, 2: Chromel–Au(Fe0.07%) thermocouples for ΔT leads, 3: Nernst voltage leads of sample, and 4: thermopower leads of reference YBCO sample.

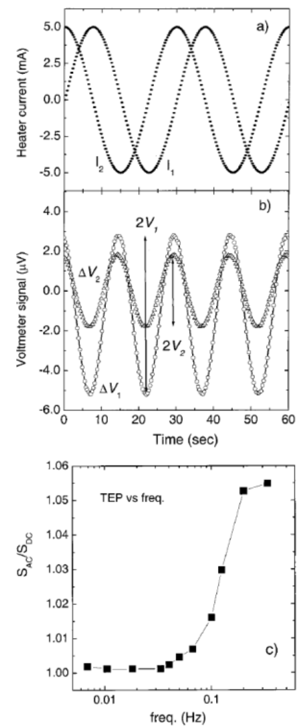

FIG. 2. (a) Heater currents and (b) $\Delta V_1(\Delta V_2)$ as a function of the time. T period of the heating cycle is 30 s and the corresponding periods of osc lation of temperature gradient and thermopower signal are 15 s. (c) S_{ac}/S vs frequency method used to determine the optimum frequency range whe $S_{ac}/S_{dc} \approx 1$ for the TEP measurements.

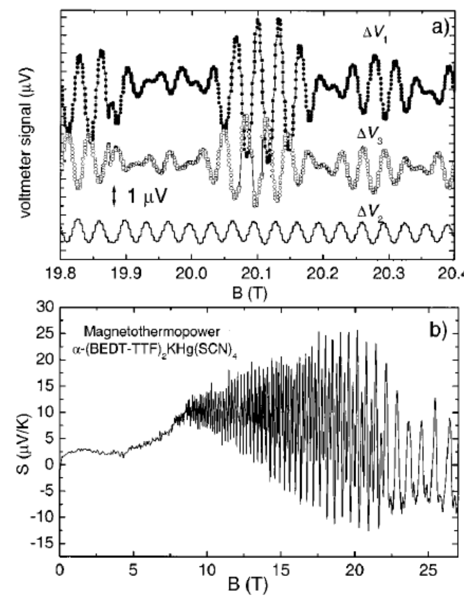

FIG. 4. Magnetothermopower. (a) ΔV_1, ΔV_2, and ΔV_3 curves under magnetic field for α-(BEDT–TTF)$_2$KHg(SCN)$_4$ at $T = 0.7$ K. (b) Derived magnetothermopower results. Note the narrow range of field in (a), which corresponds to only a few quantum oscillations in (b).

Thermopower under pressure

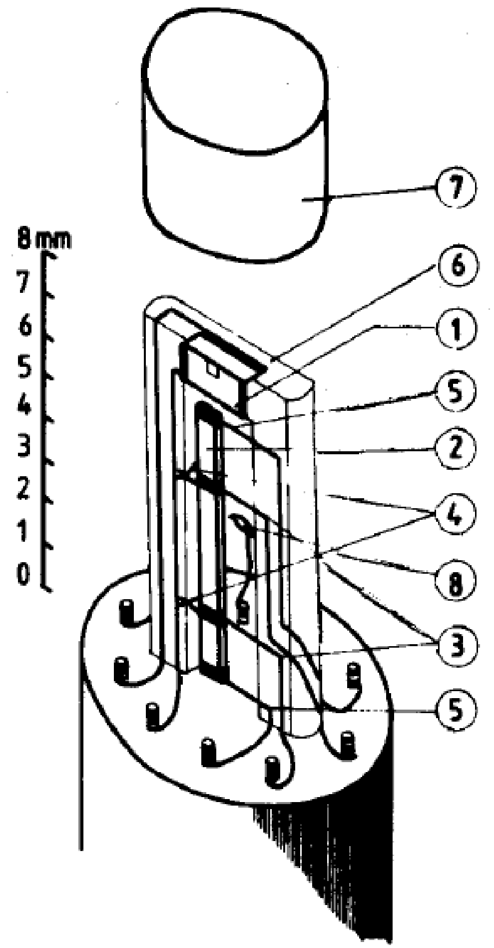

FIG. 1. Schematic representation of the apparatus for thermopower measurements under pressure; ① heater, ② sample, ③ ΔV voltage drop, ④ constantan–chromel–constantan differential thermocouple, ⑤ gold wires, ⑥ gibbet, ⑦ "chapeau," ⑧ copper–constantan thermocouple.

Both
Type E
and AuFe-Au thermocouples
are not very
sensitive to pressure

Pressure medium establishes
Thermal gradient,
Prohibitive to materials with
very high thermal conductivity

Thermopower: Magnetic polarons

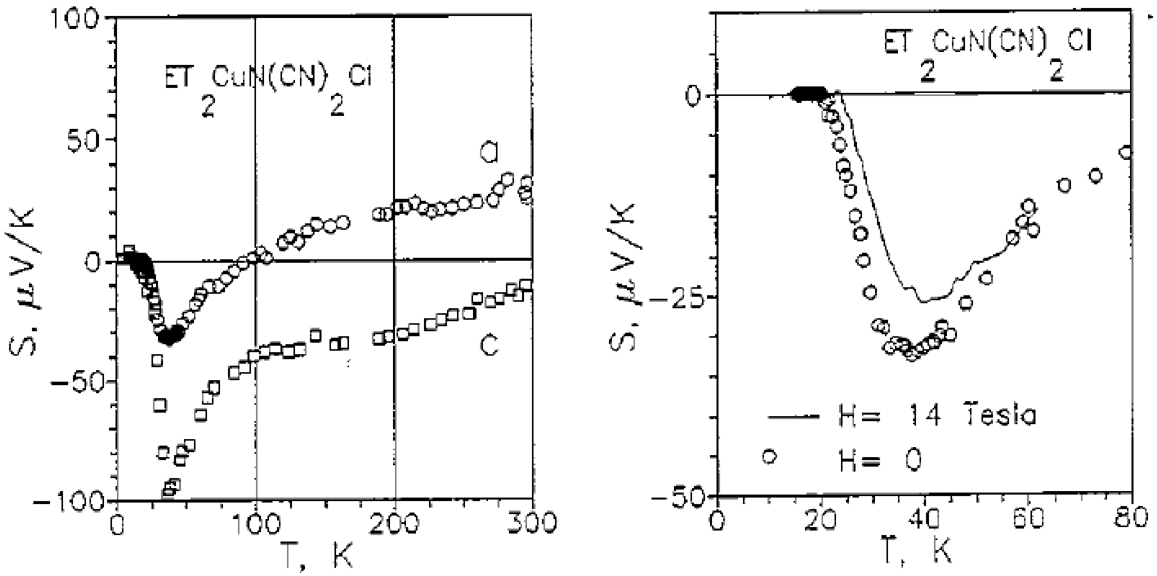

Fig.1. General view and magnetic field effect on the S vs T dependences for $ET_2Cu[N(CN)_2]Cl$

Thermopower: Tallon formula

PHYSICAL REVIEW B VOLUME 51, NUMBER 18 1 MAY 1995-II

Generic superconducting phase behavior in high-T_c cuprates: T_c variation with hole concentration in $YBa_2Cu_3O_{7-\delta}$

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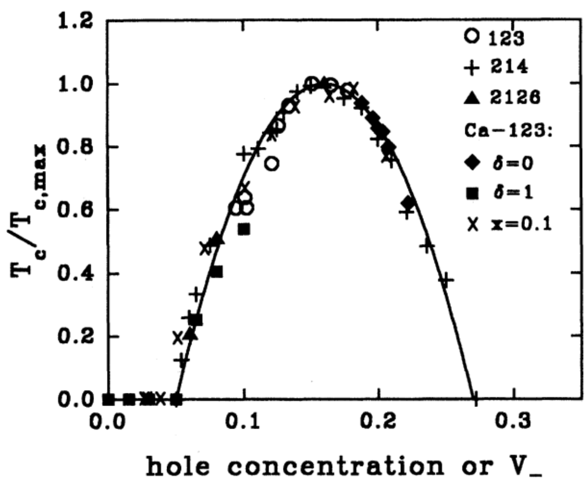


FIG. 2. T_c, normalized to $T_{c,\max}$, plotted as a function of hole concentration, p determined (i) from $p=x/2$ for $Y_{1-x}Ca_xBa_2Cu_3O_6$ (solid squares), (ii) from $p=V_-$ for $YBa_2Cu_3O_{7-\delta}$ with different δ (open circles), (iii) from $p=V_-$ for $Y_{1-x}Ca_xBa_2Cu_3O_{7-\delta}$ with $\delta\approx0.04$ and different x (solid diamonds), and (iv) from $p=V_-$ for $Y_{1-x}Ca_xBa_2Cu_3O_{7-\delta}$ with $x=0.1$ and different δ (crosses, $\times$). The solid curve is Eq. (1), the "plus" symbols (+) are T_c vs x data for $La_{2-x}Sr_xCuO_4$ and solid triangles for $La_{2-x}Sr_xCaCu_2O_6$.

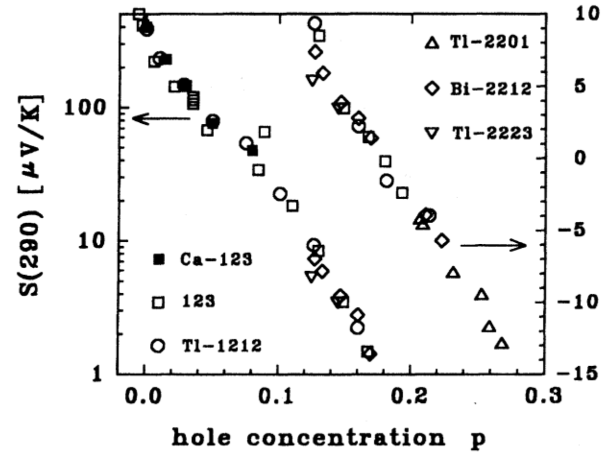


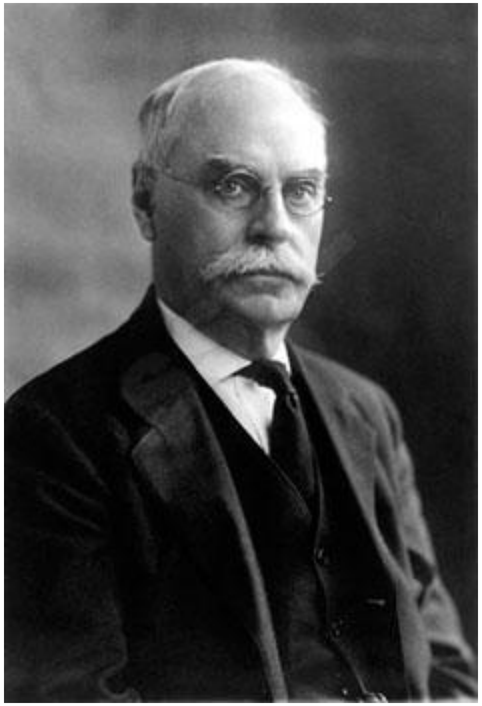
FIG. 3. Room-temperature thermoelectric power plotted as a function of hole concentration for various HTSC's as reported in Ref. 8 and for oxygen-deficient ($\delta\approx0.98$) $Y_{1-x}Ca_xBa_2Cu_3O_{7-\delta}$ for which $p=x/2$. The underdoped side has a logarithmic scale and the overdoped side a linear scale.

$$\begin{aligned} S_{290} &= 372\ \exp(-32.4p) && \text{for } 0.00<p<0.05, \\ S_{290} &= 992\ \exp(-38.1p) && \text{for } 0.05<p<0.155, \quad (2) \\ S_{290} &= -139p+24.2 && \text{for } p>0.155. \end{aligned}$$

Advantage: all transport varies with doping,
Only S does not depend on geometric factor!

Henry A. Rowland at
Johns Hopkins University

You can do anything
with cats



1848-1901

1855-1938

Proved that magnetic field is an effect of an electrically charged body in motion

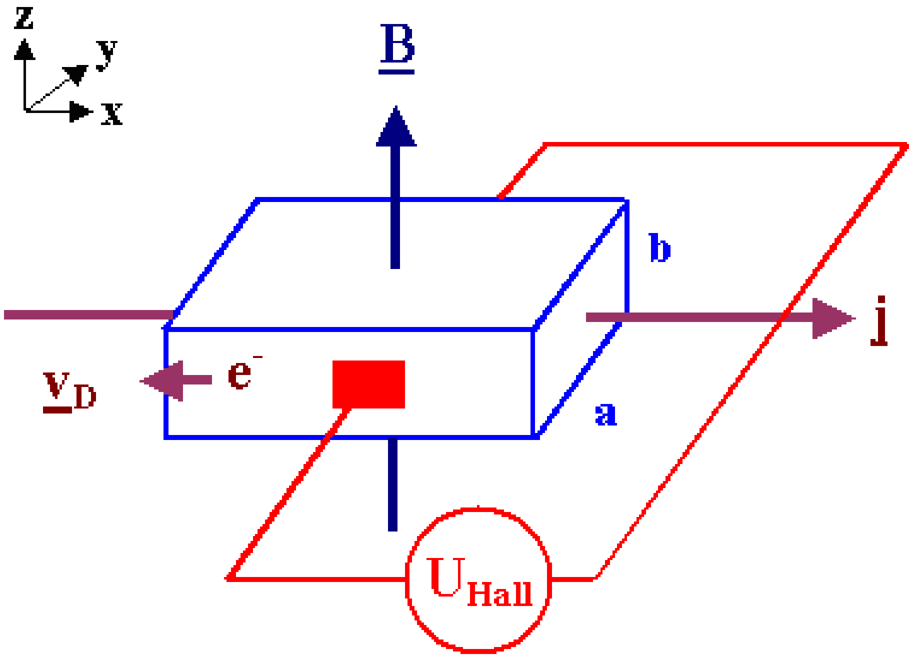
1879 Hall effect discovered

Hermann von Helmholtz student

THE HALL EFFECT HISTORY:
Hall read in his E and M textbook, written by Maxwell, that the Lorentz force, acted on the conductor and not on the charge itself.

Rowland: the charges in a metal are positive or negative?
Are they particles at all or something like a fluid or heat?

A magnetic field ***B*** is employed perpendicular
to the current direction ***j***,
a *potential difference* (i.e. a *voltage*) develops at
right angles to
both vectors.

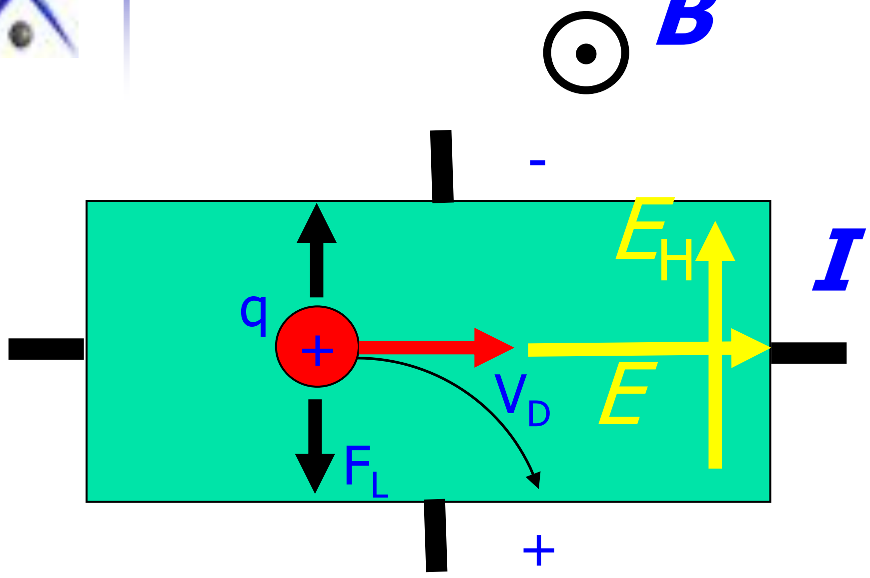


A **Hall voltage** $\boldsymbol{U_{Hall}}$
is measured perpendicular
to ***B*** and ***j***

Charge moving in magnetic feild

$$F_L = q \cdot (v_D \times \underline{B})$$

$\mathbf{v_D = \mu \cdot \mathit{E},}$
μ = mobility of the carriers



$$\sigma = qn\mu$$

$$U_H = R_H \frac{IB}{d}$$

$$R_H = \frac{1}{qn}$$

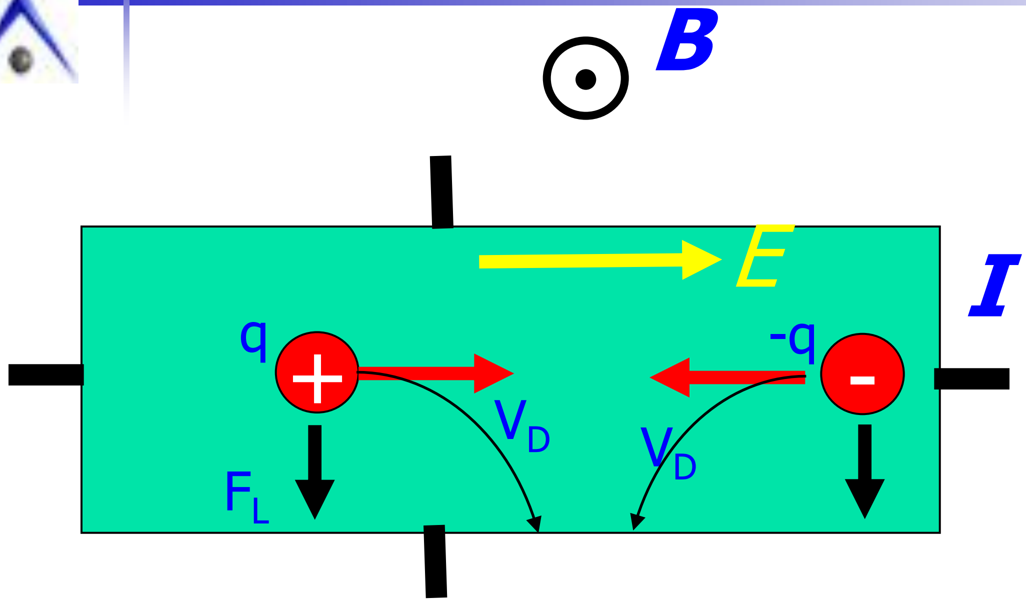
$qE_H = qV_D B$ $U_H = E_H w$

$E_H = \mu E_L B$

Here q is elementary charge ±electron

Good for:

- Sign of charge carriers
- Concentration and mobility of charge carriers (in combination with resistivity)
- Hall magnetic field sensors (one of the most precise and linear)
- Anomalous Hall effect is used to detect magnetic transitions



$$\sigma_h = qn\mu_h$$
$$\sigma_e = qn\mu_e$$

$$R_{Hh} = \frac{1}{qn_h}$$

$$R_{He} = -\frac{1}{qn_e}$$

$$R_H = \left(\frac{\sigma_h}{\sigma_e + \sigma_h}\right)^2 R_{Hh} - \left(\frac{\sigma_e}{\sigma_e + \sigma_h}\right)^2 R_{He}$$

$$\boxed{R_H = \sum_i \left(\frac{\sigma_i}{\sum \sigma_i}\right)^2 R_{Hi}}$$

Limitations:

- Hall effect is a quantitative tool at low fields where it is linear
- In materials with simple and well understood band structure
- When you have no magnetism

Other cases: still useful tool, if you understand limitations!
Difficult to make definite statements

Measurement of Hall effect

Expected values

$$R_H = \frac{1}{qn}$$

Typical numbers

Metals

$n \sim 10^{28}\ m^{-3}$

$R_H \sim 10^{-9}\ m^3C^{-1}$

I=1 mA, B=1 T, d=0.1 mm

$$U_H(Volts) = 10^{-9}m^3C\frac{10^{-3}A \times 1T}{10^{-4}m} \sim 10^{-8}V$$

U~10-100 nV

semiconductors

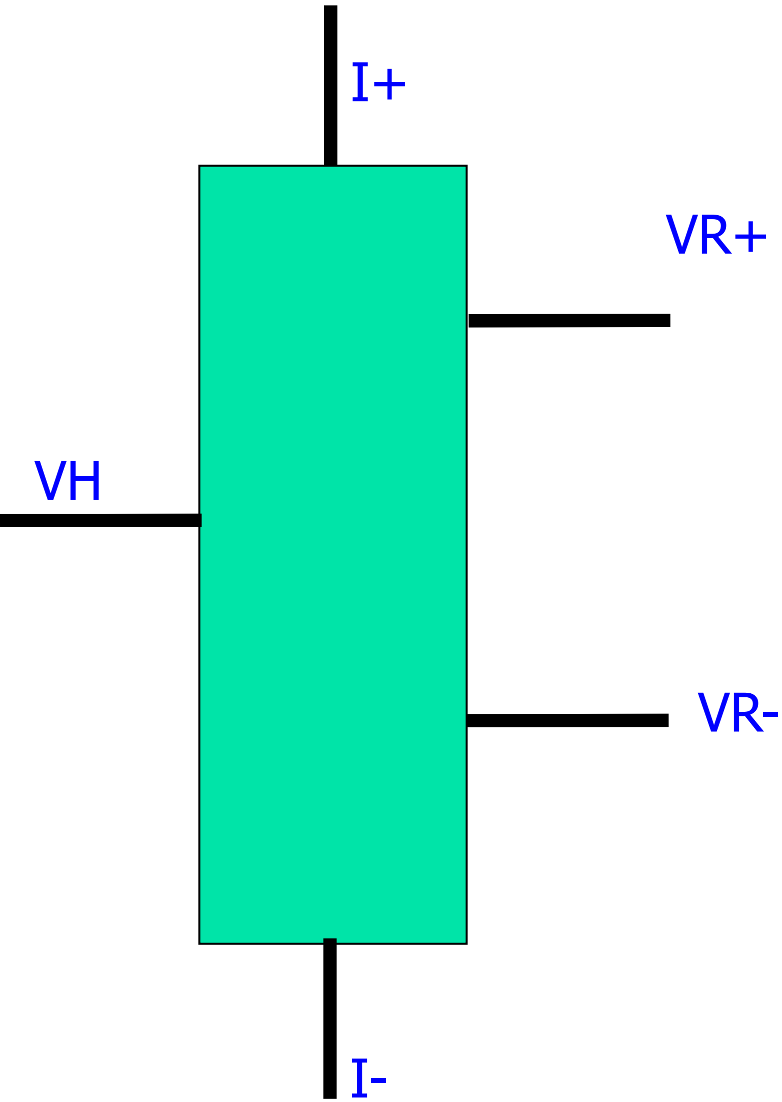
$n \sim 10^{16}\text{-}10^{24}\ m^{-3}$

$R_H \sim 10^{3}\text{-}10^{-6}\ m^3C^{-1}$

U~0.1 mV-1 V

Two very different measurements!

Hall effect sample

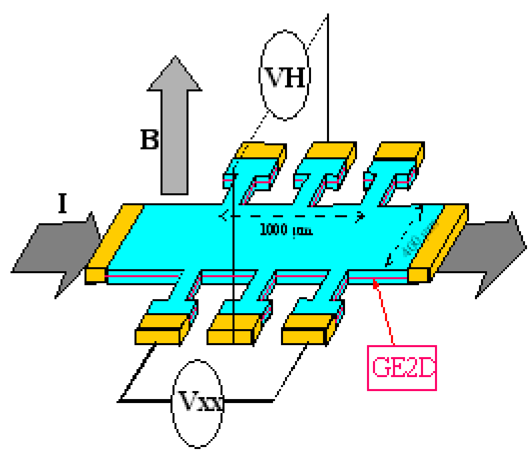


Usually measure not U_H

$$U_H = R_H \frac{IB}{d}$$

but Hall resistance $Rxy=U_H/I$

$$R_{xy} = R_H \frac{B}{d}$$



Ideal sample geometry Hall bar

Measurements

Hall resistance is defined as odd part of Rxy in field

$R_{xy}=R_{nonequipotential}+R_{MR}+U_H/I$

Measurements in positive and negative fields,
$Rxy(H)-Rxy(-H)=2U_H/I$

Ideal case: fixed temperature +H to –H sweep
In reality frequently 2 fixed fields if want T-dependence
Time consuming

Strict requirements for T-drift, $Rxy(\delta T) \ll Rxy(H)$

Option: sample rotation in magnetic field (I used in my measurements)
Quick field direction reversal, sometimes impossible (magnetic, SC samples)

Exotic ways to measure:
Double frequency modulation, use AC magnetic field and AC current

$$B = B_0 \sin(\omega_B t)$$

$$I = I_0 \sin(\omega_I t)$$

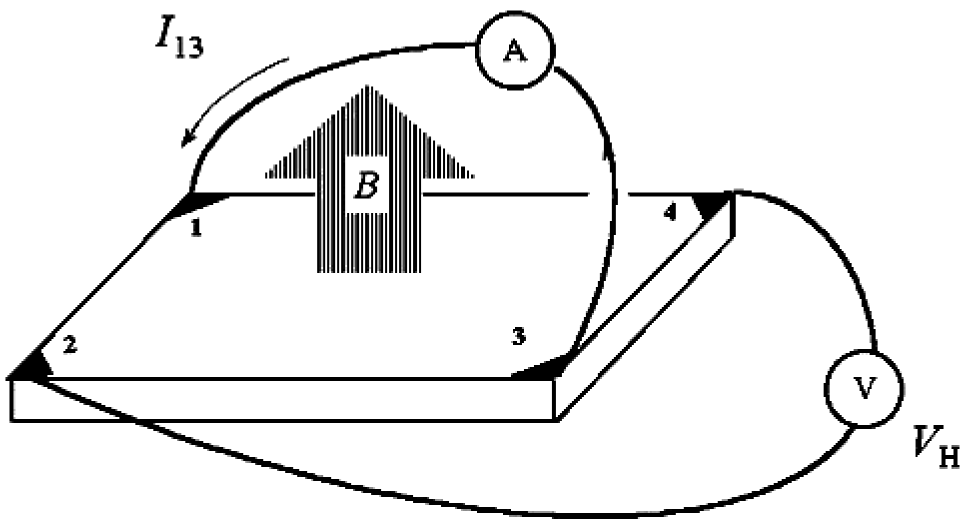
$$V = V_0 \sin[(\omega_I \pm \omega_B)t]$$

Very good but sophisticated technique
demanding for EM interference

Several text books on methods of measuring Hall effect

Van der Pauw technique

$$V_H = V_{24P}$$



Canonical behavior
"good scattering"

Metals
R_H=const(T)
Semiconductors
R_H=exp(1/T)

Anomalous behavior
Is found quite frequently

Cuprates
R_H=1/T

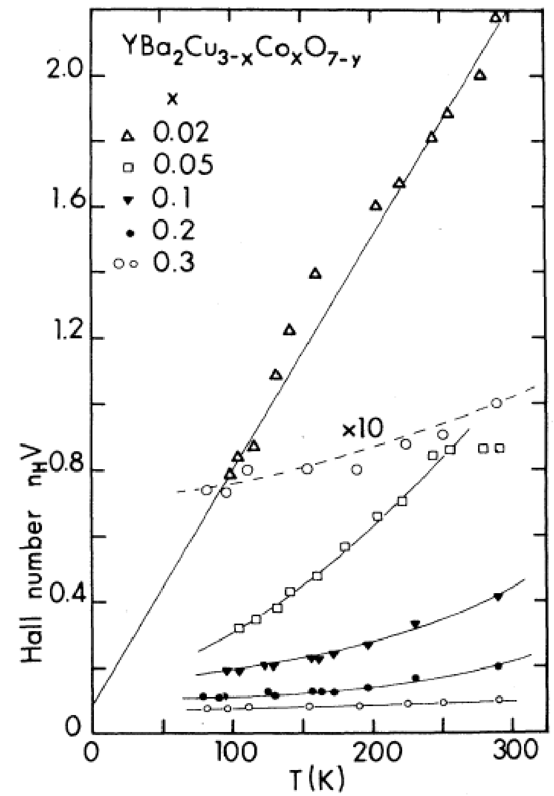


FIG. 1. Variation of the Hall number (defined as V/R_He where V=unit-cell volume, 175 Å^3) with temperature in Co-doped $YBa_2Cu_3O_7$. The data for $x=0.3$ (open circles) is replotted enlarged by a factor of 10 to show the weak T dependence. The slopes rapidly decrease as x increases. Lines are drawn to guide the eye. See Refs. 10 and 11 for T_c vs x.

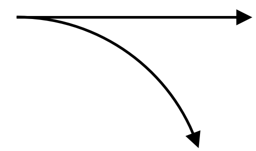
Weak field magnetoresistance: cyclic motion of electron in B

$l << R_{cyclotron}$
$\omega_c \tau << 1$

$$\sigma = qn\mu$$

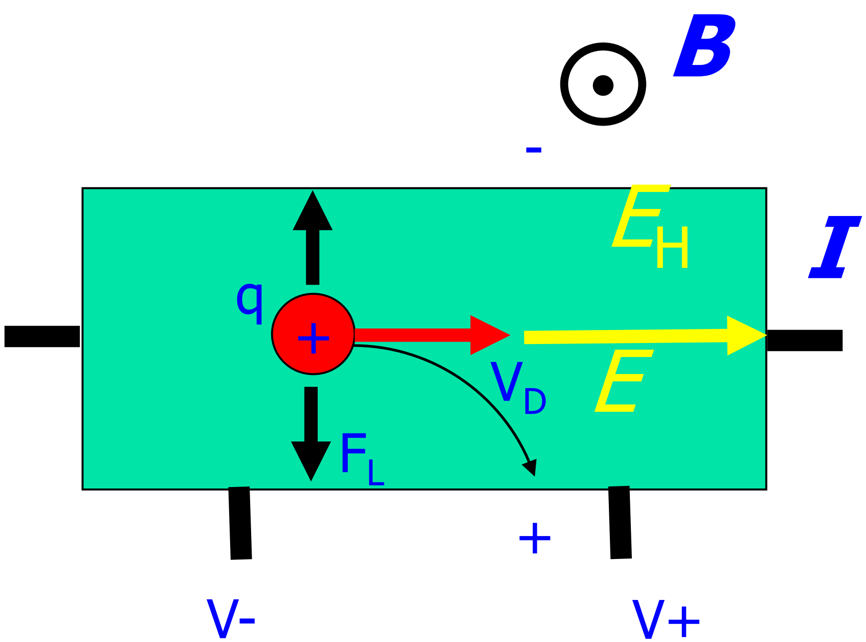
Assume

l=const

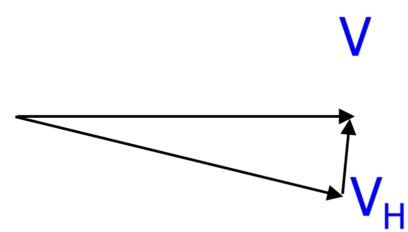


Cyclotron orbit

Transverse MR $\Delta\rho_{\perp} = \rho(H) - \rho(0)$
Usually use $\Delta\rho / \rho(0)$
Typical value
in metals $\Delta\rho / \rho(0) \sim 10^{-4}$ in 1T
Can be as big as 2 in Bi



Projection on current direction gets shorter, resistivity increases



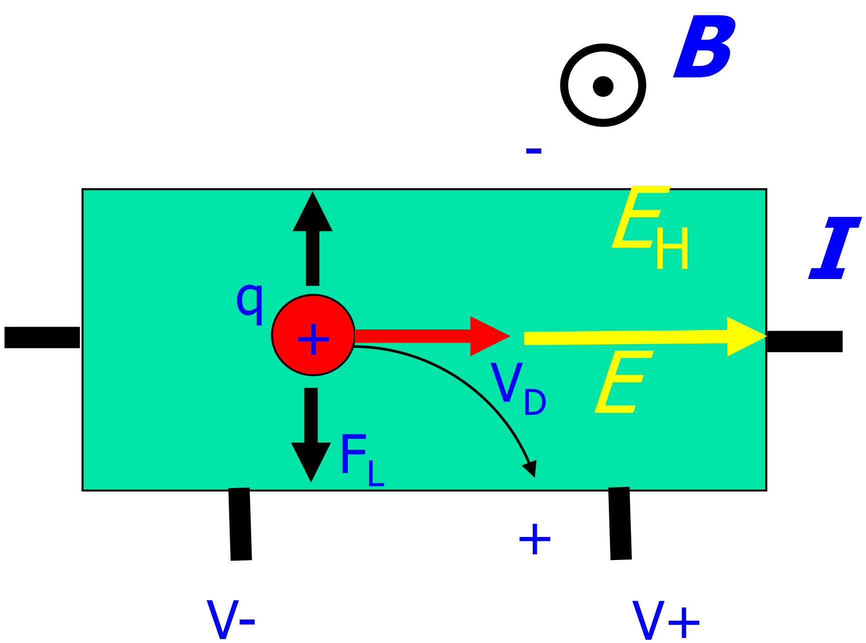
This is equivalent to Hall angle

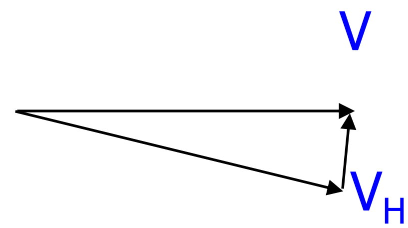
$V_H \sim H$

Actually $\rho \sim H^2$
Why?

Weak field magnetoresistance

$$\sigma = qn\mu$$





This is equivalent to Hall angle

$V_H \sim H$

The trajectory is cyclotron orbit until Hall voltage sets in

Transverse electric field makes it straight again

If no cyclotron motion, why MR?

Distribution of velocities.

$$\sigma = qn\mu$$

Hall field compensates only average velocity V_D

Hot and cold carriers still have bent orbits

Effect becomes second order ~H^2, much weaken than it could be

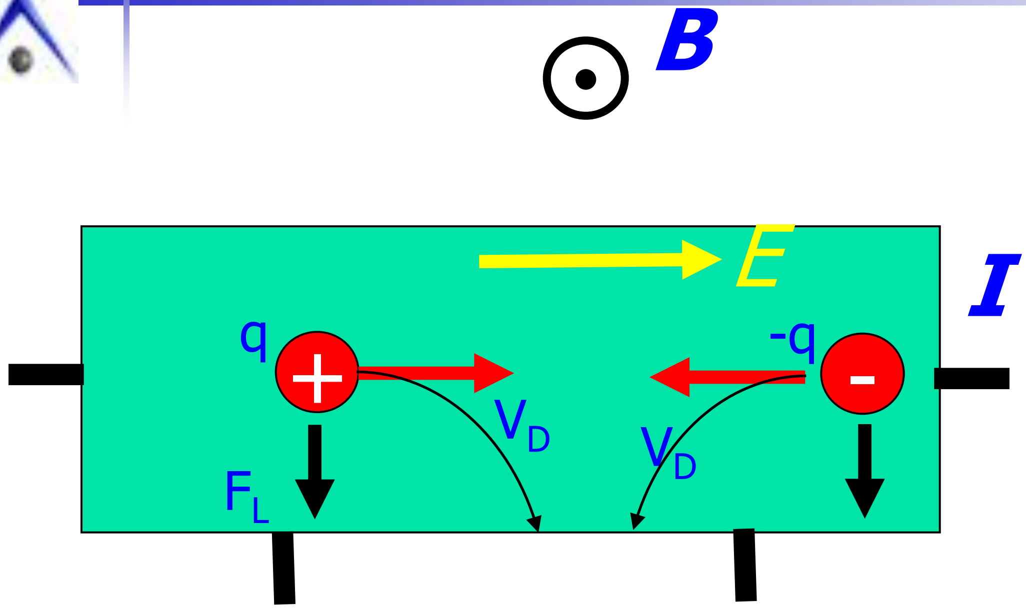
Kohler rule

For "good scattering" transverse MR is universal function

$$\frac{\Delta\rho}{\rho_0} = F(H_{ef}) \qquad H_{ef} = H\frac{\rho(T)}{\rho_0}$$

The data for same metal are on the same line

H_{eff} is actually the measure of m.f.p./$R_{cyclotron}$



$$\sigma_h = qn\mu_h$$
$$\sigma_e = qn\mu_e$$

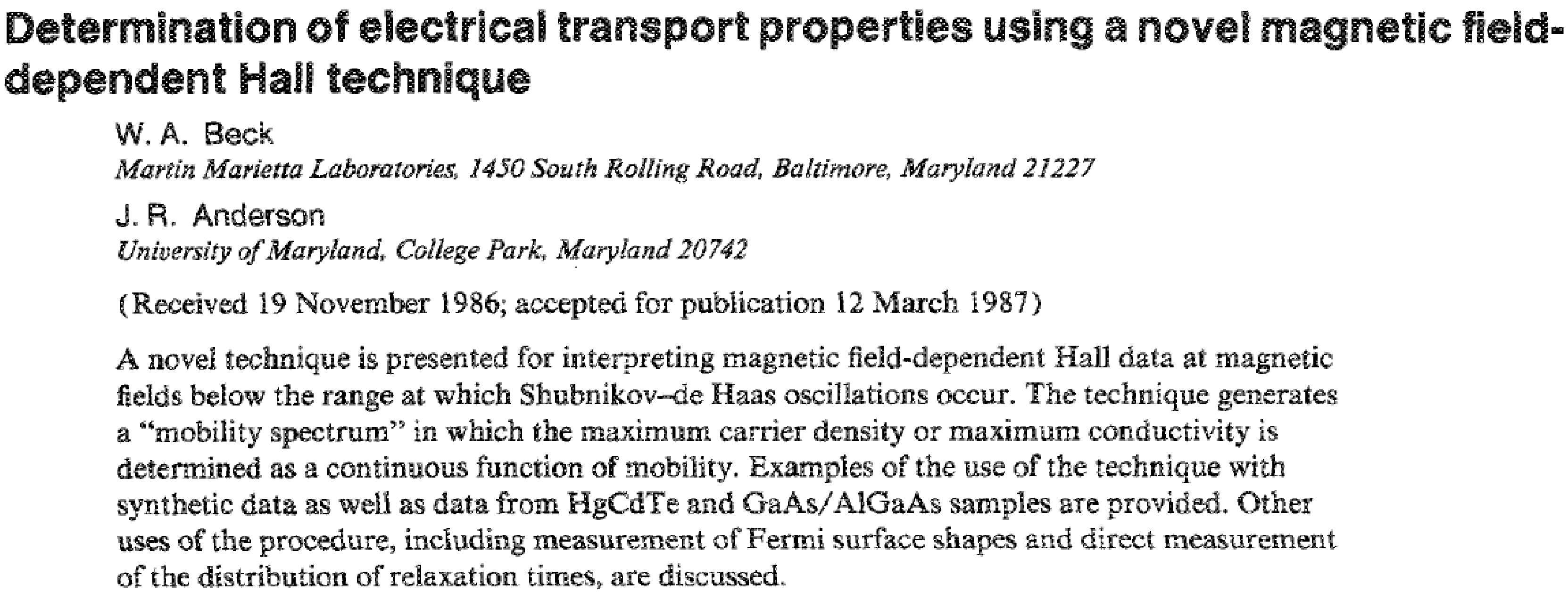
Imagine ideal compensation, V_H=0
No action of Hall voltage, all trajectories are bent

Much larger MR

Hall effect can become B-dependent (non-linear) if two carrier types have different mobilities

This is why Bi has so big MR

Mobility spectrum analysis technique


Determination of electrical transport properties using a novel magnetic field-dependent Hall technique

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(Received 19 November 1986; accepted for publication 12 March 1987)

A novel technique is presented for interpreting magnetic field-dependent Hall data at magnetic fields below the range at which Shubnikov–de Haas oscillations occur. The technique generates a "mobility spectrum" in which the maximum carrier density or maximum conductivity is determined as a continuous function of mobility. Examples of the use of the technique with synthetic data as well as data from HgCdTe and GaAs/AlGaAs samples are provided. Other uses of the procedure, including measurement of Fermi surface shapes and direct measurement of the distribution of relaxation times, are discussed.

Instead of making assumptions on number of carriers and their mobility, assume these as variables. Analysis analogous to Fourier transformation

Need relatively big magnetoresistance to apply

P. M. Chaikin, An introduction to thermopower for those Who Might want To use it to Study Organic Conductors and Superconductors
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